Review of "Variational and Extremum Principles in Macroscopic Systems", edited by Stanislaw Sieniutycz and Henrik Farkas

Given a dissipative system, is there a variational theory describing it? A significant part of the book is devoted to this question, though other aspects of variational principles are covered as well. The first half of the book covers theory, while the second presents a wide range of applications of variational principles.

Variational principles are more commonly associated with conservative systems. An example of a classic dissipative system in variational guise might thus be in order. Consider the motion of a particle in a potential field under under the influence of linear friction,

$$m\ddot{x} = -DV(x,t) - \lambda m\dot{x}. \tag{1}$$

Equation (1) can easily be seen to be equivalent to the Euler-Lagrange equation for Bateman's Lagrangian

$$L(x,\dot{x},t) := \left(\frac{m}{2}\dot{x}^2 - V(x,t)\right)e^{\lambda t}.$$

For a scientist studying a dissipative system, finding a variational principle is a dream, as powerful tools from the Calculus of Variations and numerical analysis become available. In practise, the quest for variational formulations for dissipative systems seems more like a nightmare: our understanding is not nearly as satisfactory and coherent as we would like it to be. The book makes a valuable contribution by presenting various recent approaches side by side, giving equal voice to each of them. Maugin and Kalpakides establish a variational form for classical thermoelasticity of anelastic conductors. The key ingredient here is the inclusion of thermacy to the list of fields; thermacy is the time primitive of the temperature. Anthony sketches the Lagrangian formalism for irreversible thermodynamics. Here, in analogy to quantum mechanics, complex-valued field variables are introduced, such as the field of thermal excitation, from which the absolute temperature can be derived as the squared modulus. Anthony focuses in particular on the Second Law of Thermodynamics and the Principle of Least Entropy Production. Different ways of creating Lagrangians for dissipative systems are due to Sieniutycz and Berry (based on symmetry principles and conservation laws), and Gambár and Márkus. The book is a good reference for these results, and contains papers of most of these authors. For example, Gambár's and Márkus' contributions to the book include a discussion of action principles for parabolic equations and the quantisation procedure for heat conduction. Wagner discusses the derivation of Lagrangians for hydrodynamic systems with linear damping, exploiting an analogy between hydrodynamics and the hydrodynamic picture of quantum theory.

Variational principles also appear in very different situations: for example, the speed of a travelling wave for the reaction-diffusion equation can under suitable assumptions be characterised in a variational manner. Benguria and Depassier give a very readable review, including motivations and proofs, of some results in this direction.

It is, within the space of a review, not possible to do justice to all contributions. Suffice it to say that a wide range of topics is covered in the part discussing theory, including relativistic aspects.

Both theory and applications are given about the same weight and space. The second half of the book is devoted to applications and starts with a short section of statistical physics and thermodynamics. Alternatives are presented to Jaynes' approach of statistical mechanics (the Maximum Entropy Method, where the probability density function is derived from the entropy through extremisation). Plastino, Plastino and Casas show that thermostatistics can equally be derived via a constrained extremisation of Fisher's information measure. A clear overview of a different line of thought is given by de Almeida. The entropy is derived here from the probability density as a consequence of the second law of thermodynamics. Based on data relating the Reynolds number and entropy production, Paulus and Gaggioli hypothesise that, for a specified mass flow rate, a stable fluid flow corresponds to a maximal rate of entropy production, while a stable flows corresponds for a specified pressure drop to a minimal entropy production rate.

Presumably, many readers will not be familiar with variational principles in ecology and economics. Jørgensen's paper may serve as an introduction to variational principles in ecology. The fundamental concept discussed here is ecoexergy, which measures the distance from thermodynamic equilibrium. It is argued that maximisation of ecoexergy occurs in ecosystems; however, different maximisation criteria such as emergy are discussed as well.

These sketches should give a flavour of the applications discussed; the range also includes fluid mechanics (e.g., drag in linear hydrodynamics), continuum mechanics (e.g., local and global buckling of elastic plates with composite structure), and transport phenomena. The mathematical methods employed here are often classical tools from Engineering Mathematics. The emphasis is on the derivation of variational descriptions for the applications under consideration.

The book sets out to serve as a reference for researchers in applied mathematics, macroscopic physics and chemistry as well as a source for graduate students in related fields. I feel that both goals have been accomplished. Occasional imprecision, such as the lack of a clear distinction between minimal and stationary principles in some contributions, do not destroy the overall impression that this is a thoughtfully selected and carefully edited compilation. Many of the mathematical concepts presented in this collection of papers (Hamiltonian and Lagrangian formalism, Noether's theorem) are available in a concise form in standard textbooks. The valuable contribution of the book resides in the presentation of recent developments, particularly of variational theories for dissipative systems, as well as the wealth of applications. This should make the study of the book under review a worthwhile endeavour for students. The breadth of topics and approaches under discussion recommends the book to scientists and applied mathematicians alike.

Johannes Zimmer, Department of Mathematical Sciences, University of Bath, UK